Int. J. Heat Mass Transfer. Vol. 31, No. 2, pp. 241–244, 1988
Printed in Great Britain

0017–9310/88 $3.00 + 0.00
Pergamon Journals Ltd.

Comparison of entropy generation and conventional method of optimizing a gas turbine regenerator

DAVID S. HERBEIN and WARREN M. ROHSENOW
Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

(*Received* 8 *June* 1987)

Abstract—A gas turbine counterflow plate-finned regenerator is sized for (a) a minimum volume at a given cycle efficiency degrade $(\eta_\infty - \eta)$ which is the same magnitude as entropy generation, N_{ST}, and (b) the volume resulting in minimum generation, N_{ST}, along a path of constant mass velocity, G, as suggested by Bejan (*ASME J. Heat Transfer* **99**, 374 (1977)). This latter procedure results in much larger regenerators for any $(\eta_\infty - \eta)$ or N_{ST}.

INTRODUCTION

THE ENTROPY generation concept has been developed in recent years for suggesting the optimum design of various power plant components. Here, we show the comparison of the results of the optimum design of a gas turbine regenerator by the previously used 'conventional method' and the entropy generation method.

CYCLE CONSIDERED

The regenerator gas turbine cycle analyzed here is shown in Fig. 1. The ideal cycle, 1–2–3–4–5–6, has a fictitious ideal regenerator with 100% effectiveness and zero pressure drop. Its cycle efficiency is calculated to be $\eta_\infty = 57.6\%$ and 260 kW.

The real cycle will employ a counterflow plate-finned regenerator with surface 2.0, Fig. 10-19 of Kays and London [1], with aluminum fins on both hot and cold sides. This cycle is represented by 1–2–3′–4′–5′–6′ with finite pressure drop and regenerator effectiveness less than 1.0. For this example no pressure drop is assigned to the combustion chamber nor ducting.

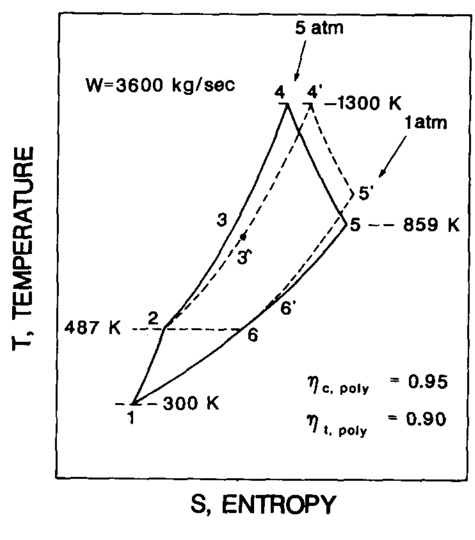

FIG. 1. Regenerative gas turbine cycle.

CONVENTIONAL METHOD

Constant air properties were used and the wc_p on both sides of the generator were the same. Then, cycle efficiency is

$$\eta = \frac{(i_{4'} - i_{5'}) - (i_2 - i_1)}{(i_{4'} - i_{3'})} \tag{1}$$

and regenerator effectiveness is

$$\varepsilon = \frac{T_{3'} - T_2}{T_{5'} - T_2}. \tag{2}$$

It is readily shown that for small $\Delta P/P$, the loss in turbine work is

$$\frac{\Delta W}{w\eta_T c_p T_{5'}\gamma} = \left(\frac{P_c}{P_h}\right)\gamma\left(\frac{\Delta P_c}{P_c} + \frac{\Delta P_h}{P_h}\right). \tag{3}$$

This shows the loss in turbine work is proportional to $\Sigma(\Delta P/P)$ regardless of the distribution between the hot and the cold side of the regenerator. In a counterflow regenerator, the ratio of $(\Delta P/P)_c/(\Delta P/P)_h$ is determined by the ΔP equations and $(\Delta P/P)_c \ll (\Delta P/P)_h$. The regenerator is sized by the calculation procedure in Kays and London [1].

The results of the calculations are shown in Fig. 2. The dashed curves are lines of constant degradation of cycle efficiency $(\eta_\infty - \eta)$. The minimum volume at a given $(\eta_\infty - \eta)$ or N_{ST} are shown as open circles. The solid line in Fig. 3 is a plot of these minimum volumes at a given $(\eta_\infty - \eta)$. Also shown are the magnitudes of

NOMENCLATURE

c_p	specific heat at constant pressure
f	friction factor
G	mass velocity
h	heat transfer coefficient
i	specific enthalpy
L	flow length
N_{ST}	number of entropy generation units, total
NTU	number of heat transfer units
P	pressure, absolute
$\Sigma\Delta P/P$	summation of hot and cold pressure losses
Pr	Prandtl number
r_h	hydraulic radius
R	ideal gas constant
s	specific entropy
St	Stanton number
T	temperature, absolute
V	core volume
w	mass flow rate.

Greek symbols

Δ	difference
ε	heat exchanger effectiveness
γ	ratio of specific heats, c_p/c_v
η	cycle efficiency with losses
η_∞	cycle efficiency without losses
η_{pc}	polytropic compressor efficiency
η_{pt}	polytropic turbine efficiency
ρ	gas density
τ	temperature span parameter.

Subscripts

c	cold side
h	hot side.

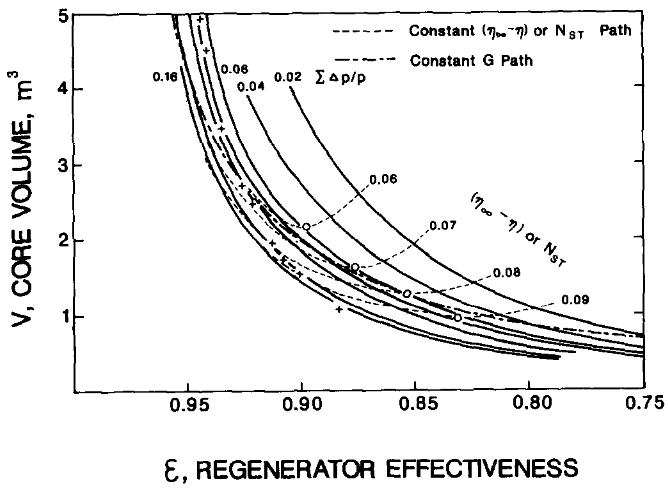

FIG. 2. Regenerator core volume.

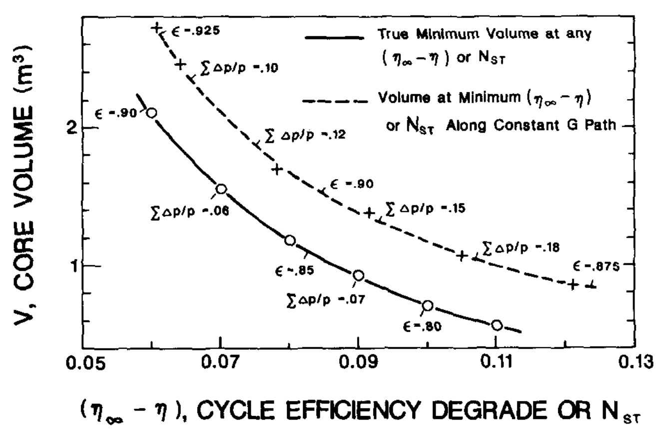

FIG. 3. Comparison of two methods of optimizing regenerator volume.

ε and $\Sigma(\Delta P/P)$ at these minimum volumes. It is noted that the magnitudes of N_{ST} are essentially the same as $(\eta_\infty - \eta)$.

ENTROPY GENERATION METHOD

For a gas turbine regenerator with wc_p the same on both sides, Bejan [2] shows the entropy generation expression as follows:

$$\left.\begin{aligned} N_{s,c} &= \left(\sqrt{\left(\frac{T_{5'}}{T_2}\right)} - \sqrt{\left(\frac{T_2}{T_{5'}}\right)}\right)^2 \frac{1}{NTU_c} + \frac{R}{c_p}\left(\frac{\Delta P}{P}\right)_c \\ N_{s,h} &= \left(\sqrt{\left(\frac{T_{5'}}{T_2}\right)} - \sqrt{\left(\frac{T_2}{T_{5'}}\right)}\right)^2 \frac{1}{NTU_h} + \frac{R}{c_p}\left(\frac{\Delta P}{P}\right)_h \end{aligned}\right\} \quad (4)$$

$$N_{ST} = N_{s,c} + N_{s,h}.$$

Substituting expressions for NTU and ΔP, Bejan [2] shows

$$N_{s,h\,or\,c} = \frac{\tau}{(L/r_h)St} + \frac{R}{c_p} f \frac{L}{r_h} \frac{G^2}{2\rho P} \quad (5)$$

for each side, where

$$\tau = \left(\sqrt{\left(\frac{T_{5'}}{T_2}\right)} - \sqrt{\left(\frac{T_2}{T_{5'}}\right)}\right)^2$$

$$St = \frac{h}{c_p G}.$$

Substituting equation (5) into equation (4) for both sides and setting $dN_{ST}/dL = 0$ results in

$$\left(\frac{L}{r_h}\right)_{\min N_{ST}} = \frac{\tau/St}{\frac{fR}{c_p}\frac{G^2}{2}\frac{1}{2}\left(\frac{1}{\rho_c P_c} + \frac{1}{\rho_h P_h}\right)} \quad (6)$$

which is the L/r_h for minimum N_{ST} for a counterflow regenerator with equal wc_p, uniform properties, same plate-finned surface on each side following a path of constant G. To obtain equation (6) from equation (5), τ, St, f, ρ, and P are taken as approximately constant along the G path.

Calculations were made for various magnitudes of $\Sigma(\Delta P/P)$ and ε with L being established by equation (6). This requires varying τ, St, f, and G in equation (6). For each case, magnitudes of N_{ST}, volume, and $(\eta_\infty - \eta)$ are calculated. The volumes at this minimum N_{ST} along the constant G path are shown as crosses on Fig. 2 and as a dashed curve in Fig. 3. These volumes are much greater than the minimum volumes at any given N_{ST} or $(\eta_\infty - \eta)$.

Also shown on Fig. 2 as a dash-dot line is the locus of generator volumes along one constant G path which is a constant frontal area path with varying regenerator lengths. Along this dash-dot curve, $(\eta_\infty - \eta)$ or N_{ST} decreases to a minimum and then increases again. The volume at this minimum N_{ST} is larger than the true minimum volume at the same N_{ST}.

Alternatively the generator could be sized along a constant length path with varying frontal area. The volume at minimum $(\eta_\infty - \eta)$ or N_{ST} along this path would also be larger than the true minimum volume at any $(\eta_\infty - \eta)$ or N_{ST}, shown by the solid curve in Fig. 3.

In the unusual situation that either length or frontal area would be dictated to be fixed, then sizing the regenerator along either of these paths would be warranted. However, these regenerators would have larger volumes than the true minimum at that $(\eta_\infty - \eta)$ or N_{ST}.

DISCUSSION

The nature of the results and conclusions drawn for the example cycle would be the same for other cycles and regenerator surfaces. For a high effectiveness regenerator, the counterflow arrangement is dictated. Here, with the same finned surface on both sides, $(\Delta P/P)_c/(\Delta P/P)_h \simeq (P_h/P_c)^2$ or approximately 1/25. If a closer plate spacing were used on the clean air cold side, more ΔP would appear on the cold side for any total $\Sigma(\Delta P/P)$. This would result in even smaller minimum volumes at any given N_{ST} or $(\eta_\infty - \eta)$.

CONCLUSIONS

(1) In selecting a gas turbine regenerator, essentially the same results are obtained either by the conventional method of minimizing degradation of cycle effectiveness $(\eta_\infty - \eta)$ or by minimizing entropy generation N_{ST}.

(2) In selecting an 'optimum' regenerator, care must be exercised in the selection of the path along which the optimization is calculated. It has been shown that the volume of the regenerator for minimum N_{ST} or $(\eta_\infty - \eta)$ along a constant G path is much greater than the true minimum volume at any $(\eta_\infty - \eta)$ or N_{ST}.

REFERENCES

1. W. Kays and A. London, *Compact Heat Exchangers*. McGraw-Hill, New York (1964).
2. A. Bejan, The concept of irreversibility in heat exchanger design: counterflow heat exchangers for gas-to-gas applications, *ASME J. Heat Transfer* **99**, 374 (1977).
3. D. Herbein, Comparison of entropy generation and conventional design methods for heat exchangers, Naval Engineer Thesis, Department of Ocean Engineering, MIT (1987).

COMPARAISON ENTRE LA GENERATION D'ENTROPIE ET LA METHODE CONVENTIONNELLE D'OPTIMISATION D'UN REGENERATEUR DE TURBINE A GAZ

Résumé—Un régénérateur à plaque ailetée, à contre-courant pour turbine à gaz est dimensionné (a) pour un volume minimal à une dégradation donnée de rendement de cycle $(\eta_\infty - \eta)$ qui est de même valeur de la création d'entropie N_{ST}, et (b) pour le volume correspondant au minimum de génération N_{ST} dans un parcours à vitesse de masse G constante, comme suggéré par Bejan (*ASME J. Heat Transfer* **99**, 374 (1977)). Cette procédure conduit à des régénérateurs plus grands pour n'importe quels $(\eta_\infty - \eta)$ ou N_{ST}.

VERGLEICH EXERGETISCHER UND HERKÖMMLICHER OPTIMIERUNGSVERFAHREN FÜR EINEN GASTURBINEN-REGENERATOR

Zusammenfassung—Ein Gasturbinen-Regenerator, der im Gegenstrom arbeitet, kann nach folgenden Gesichtspunkten ausgelegt werden: (a) Minimales Volumen bei vorgegebenem Wirkungsgradverlust $(\eta_\infty - \eta)$ für einen Zyklus—dies ist dieselbe Größe wie die Entropieerzeugung N_{ST}; (b) Volumen bei minimaler Entropieerzeugung N_{ST}—was gemäß einem Vorschlag von Bejan (*ASME J. Heat Transfer* **99**, 374 (1977)) bei konstanter Massenstromdichte G erreicht wird. Das letztgenannte Verfahren führt für sämtliche Werte von $(\eta_\infty - \eta)$ bzw. N_{ST} zu wesentlich größeren Regeneratoren.

СРАВНЕНИЕ МЕТОДА ПРИРАЩЕНИЯ ЭНТРОПИИ С ОБЩЕПРИНЯТЫМ МЕТОДОМ ОПТИМИЗАЦИИ ГАЗОТУРБИННОГО РЕГЕНЕРАТОРА

Аннотация—Газотурбинный противоточный пластинчато-оребренный регенератор рассчитывается для двух случаев: (а) минмального объема при заданном снижении эффективности цикла $(\eta_\infty - \eta)$, равной по величине приращению энтропии N_{ST}, и (б) для объема, при котором имеет место минимальное приращение энтропии N_{ST}, при постоянной массовой скорости G (согласно Бежану) (*ASME J. Heat Transfer* **99**, 374 (1977)). Этот последний случай реализуется в большинстве регенераторов для любых $(\eta_\infty - \eta)$ или N_{ST}.